Machine Learning Exploration of Flavor Structures in Froggatt-Nielsen models [arXiv : 2303.xxxxx]

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1. Abstract

- The Froggatt-Nielsen model explains the mass hierarchy of matter particles & the flavor mixtures.
- The large parameter space makes brute-force difficult.

We use Machine Learning. (notably Reinforcement Learning)

- We efficiently find sets of parameters that simultaneously reproduce the quark-lepton masses & flavor mixing.
- We show the CP symmetry of the lepton sector can be significantly violated.

2. Method

2.1. Froggatt-Nielsen Model

Yukawa lagrangian has additional $U(1)$ flavor symmetry.

$$\begin{aligned} L_{\rm Yuk} &= y_{ij}^u \left(\frac{\phi}{M}\right)^{n_{ij}^u} \bar{Q}^i H^c u^j + y_{ij}^d \left(\frac{\phi}{M}\right)^{n_{ij}^d} Q^i H d^j \\ &+ y_{ij}^\nu \left(\frac{\phi}{M}\right)^{n_{ij}^\nu} \bar{L}^i H^c N^j + y_{ij}^l \left(\frac{\phi}{M}\right)^{n_{ij}^l} L^i H l^j \\ &+ \frac{1}{2} y_{ij}^N \left(\frac{\phi}{M}\right)^{n_{ij}^N} M \bar{N}^{ci} N^j + \rm h.c. \end{aligned}$$

Yukawa couplings y are $O(1)$ parameters.

$L_{\rm Yuk}$ is $U(1)$ invariant $\leftrightarrow$ sums of $U(1)$ charges are zero
ex. $q(\phi) n_{ij}^u - q(Q^i) - q(H) + q(u^j) = 0$

When complex scalar field ϕ develops an expectation value $\langle\phi\rangle$, Froggatt-Nielsen (FN) charges will lead to a hierarchical structure of physical Yukawa couplings (ex., $Y_{ij}^u = y_{ij}^u \langle\phi\rangle^{n_{ij}^u}$).

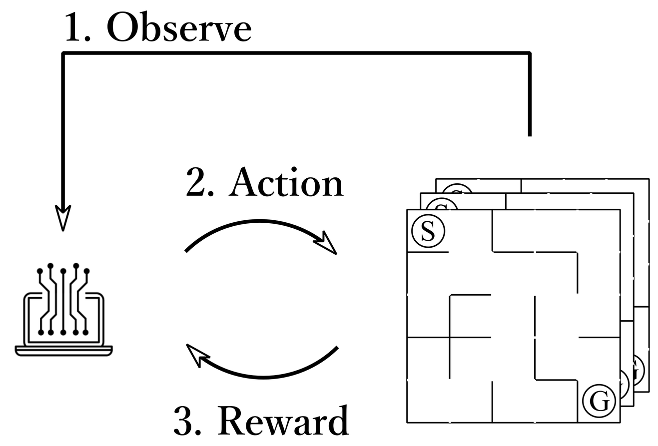
Problem) To find the appropriate parameters q & $\langle\phi\rangle$,
a vast number of combinations must be searched.
$-9 \le q \le 9 \ \rightarrow\ 19^{20} \sim 10^{25}$ patterns

We focus on the application of Machine Learning (ML).

2.2. Reinforcement Learning

Reinforcement Learning (RL) is a method of ML.

Agent
1. observes the environment
2. chooses actions
3. gets rewards



The agent autonomously find the principle of action that maximizes the total reward.

3. Setting of Learning

Agent : increasing or decreasing any FN charge of matters by ± 1 (repeat up to 32 steps)

Reward : "the masses of particles & the mixing matrix" (calculated from FN charges)

The closer those are to the experimental values, the more points the agent get.

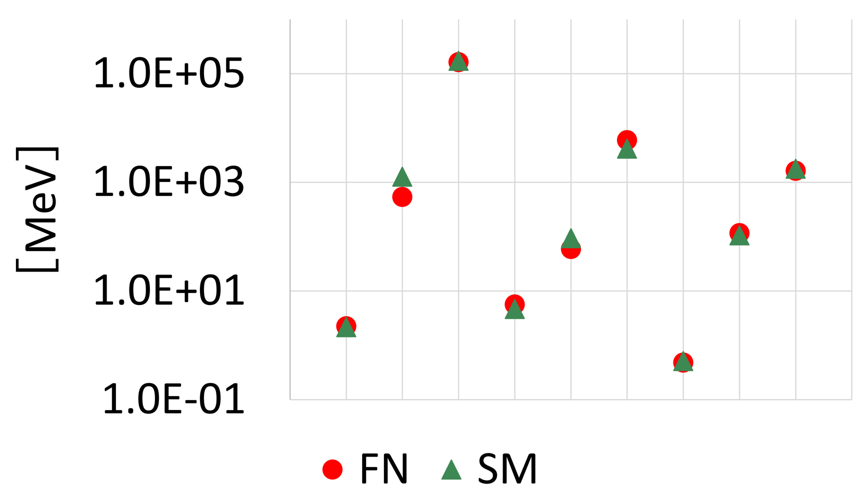
Repeat the above for 100,000 episodes...

The agent find the principle on its own to derive the FN charges that conforms to the experimental values.

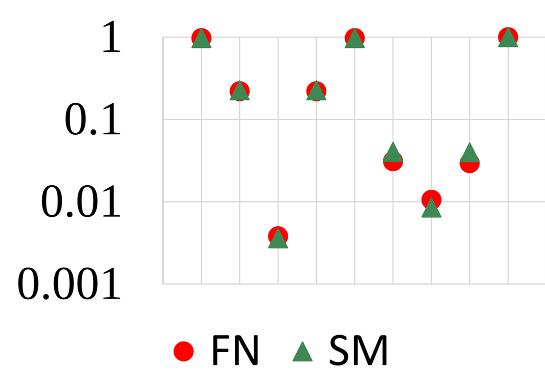
4. Result from Reinforcement Learning

In the case of normal ordering of neutrinos, $Q = \left(\begin{array}{ccc|ccc|ccc|c} Q_1 & Q_2 & Q_3 & u_1 & u_2 & u_3 & d_1 & d_2 & d_3 & H \\ -5 & -4 & -2 & 3 & 2 & 0 & -1 & -2 & -1 & 2 \\ \hline L_1 & L_2 & L_3 & N_1 & N_2 & N_3 & l_1 & l_2 & l_3 & \phi \\ -2 & -3 & -2 & 4 & 2 & 7 & -1 & -1 & 3 & -1 \end{array}\right)$

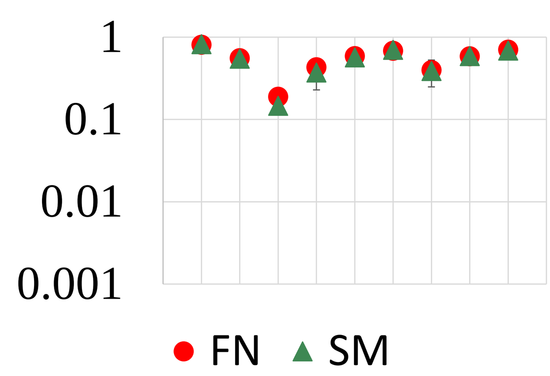
Mass Hierarchy
(u,c,t,d,s,b,e,mu,tau)



CKM elements



PMNS elements



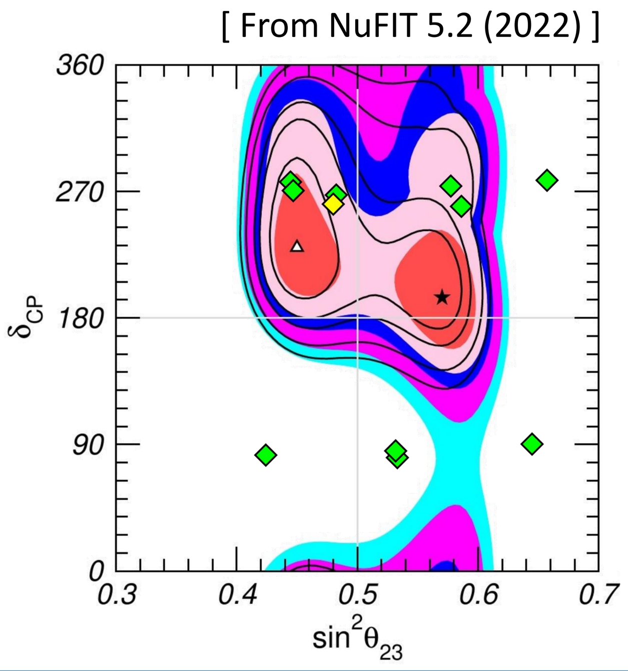
quark-lepton masses & flavor mixing are actually reproduced.

Moreover, we found the CP violation in lepton sector.
The non-zero Jarlskog invariant $\leftrightarrow$ CP is violated. (yellow)

$$J_{\rm PMNS} = -0.0423\ ,\ \delta_{\rm CP} = -0.517\pi$$

We have found other parameter sets (green), which also violate the CP in the lepton sector.

[From NuFIT 5.2 (2022)]



5. Conclusions and Future Works

1. In our work, we have also searched inverted hierarchy of neutrinos, reproducing correct mass hierarchies and mixing.
2. In both of the orders, we found non-zero Jarlskog invariant. This means that CP violation of lepton sector is violated.
3. We propose the Machine Learning (Reinforcement Learning) to search flavor models is one of effective methods.

1. This work do not derive CP violation in quark sector. We will try that ...
   1.1. extend $O(1)$ Yukawa couplings to complex.
   1.2. introduce two complex scalar fields ϕ_1, ϕ_2.
2. We fix the scale of right-handed neutrino $M = 10^{15}\rm GeV$. If this value can be changed, more precise sets of parameters may be found.
3. It is expected that Reinforcement Learning gives some predictions about neutrino mass order.